# DIFFUSION COEFFICIENT FOR THE MOTION OF A LIQUID THROUGH A PACKING 

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A description is given of a method of determining the diffusion coefficient for the motion of a liquid through a packing. The effect of the physical properties of the system on the diffusion coefficient and standard deviation are noted.

In order to describe the motion of a liquid through a packing various models are used [1-3]. The actual process is best reflected by the diffusion model $[2,3]$.

For this model the differential equation of motion in cylindrical coordinates has the form

$$
\begin{equation*}
\frac{\partial w(r, \Phi, h)}{\partial h}=D\left(\frac{\partial^{2} w}{\partial r^{2}}+\frac{1}{r} \frac{\partial w}{\partial r}+\frac{1}{r^{2}} \frac{\partial w}{\partial \Phi}\right) \tag{1}
\end{equation*}
$$

The solution of Eq. (1) with the boundary conditions

$$
\mathfrak{w}(R, 0)=0 \text { for } 0<R \leqslant 1, \lim _{H \rightarrow 0} w(0, H)=\infty
$$

is given [2] by the expression

$$
\begin{equation*}
\omega(R, H)=1+\sum_{i=1}^{n} \frac{J_{0}\left(z_{i} R\right)}{J_{0}^{2}\left(z_{i}\right)} \exp \left(\cdots z_{i}^{2} a H\right) \tag{2}
\end{equation*}
$$

At the same time, it has been noted [4] that the motion of the liquid is characterized by a high degree of nonuniformity of the irrigation density for elements of the bed cross section existing under identical conditions.


Fig. 1. Mean irrigation density over cross section w (relative units) of central tube as a function of the thickness of the packing and the diffusion coefficient $a \mathrm{H}$ (relative units).

This makes it necessary to adopt a statistical approach. The motion of water admitted along the axis of the apparatus was experimentally investigated in packings of different kinds. The diameter of the apparatus was 106 mm . The diameter of the tube for introducing the water was 4 mm , i.e., less than the diameter of the packings. This made it possible to assume that the liquid is introduced in a thin jet along the axis of the apparatus. The packing was supported on a grid of stretched wires. Underneath the bed the liquid was collected in a tube 23 mm in diameter
mounted along the axis of the apparatus and in the annular space formed by the walls of the apparatus


Fig. 2. Mean irrigation density distribution curves for various packings ( $\varphi$ is the fraction of the total flow entering the central tube, and $P$ is the probability):

1) Raschig rings $8 \times 8 \times 1.5$; 2 and 3) ceramic and polyethylene granules with $\phi 5 \mathrm{~mm}$ and $l=7 \mathrm{~mm}$.
and the walls of the tube. It is clear from Eq. (2) that the irrigation density is a function of the radius and varies with the radius; therefore when the liquid is collected in a central tube it is possible to determine only the mean value of the irrigation density for the cross section of that tube. This method of investigation permits multiple repetition of the experiments and the determination of the most probable value of the mean irrigation density for the cross section of the central tube from the results of a number of experiments.

The numerical solution of (2) for the experimental conditions ( $R=0-0.217$ for the cross section of the bed above the central tube) gives the mean irrigation density of the part of the flow that enters the central tube as a function of the height of the bed and the diffusion coefficient in the form of the graph presented in Fig. 1.

From this graph we determined the diffusion coefficient for a known bed thickness and the most probable value of the mean irrigation density obtained in the experiments.

The curves in Fig. 2 show the probability distributions of the mean irrigation density for different packings of thickness $\mathrm{h}=28 \mathrm{~mm}$. Each curve is based on the results of 50 experiments. Before each experiment the packing was shaken to change the structure of the bed. From the distribution curve we determined the most probable value of the mean irrigation density and the standard deviation.

It is clear from Fig. 2 that for a packing consisting of polyethylene granules the standard deviation is greater than for the identical ceramic granules. Reducing the surface tension of the water by introducing surface-active agent OP-10 leads to the same variation. These effects can be attributed to the influence of the wettability of the packing and surface tension.


Fig. 3. Diffusion coefficient D (cm) as a function of the thickness $h(c m)$ of the bed of ceramic granules.

For a bed consisting of balls 6 mm in diameter the results were contradictory. This may be attributable to the dense packing of this particular bed and the flow of liquid in the form of a continuous film completely filling the pores between the balls.

In the proposed method of determining the diffusion coefficient the thickness of the packing can be varied only within narrow limits, since when the diffusion coefficient is determined at the ends of the curve in Fig. 1 the accuracy of the determination is reduced.

Measurements of the diffusion coefficient made within the range of permissible thicknesses are shown in Fig. 3, from which it follows that the nondependence of the diffusion coefficient on bed thickness confirms the validity of the diffusion model as a description of the motion of a liquid through a packing.

## NOTATION

$\mathrm{w}(\mathrm{R}, \mathrm{H})=\mathrm{w}_{\mathrm{loc}} / \mathrm{w}_{\mathrm{m}}$ is the dimensionless irrigation density; $w_{l o c}$ and $w_{m}$ are the local and mean irrigation intensities; $R=r / r_{a}$ is the dimensionless radius; $\mathrm{r}_{\mathrm{a}}$ is the radius of the apparatus; $\mathrm{H}=\mathrm{h} / \mathrm{ra}_{\mathrm{a}}$ is the dimensionless thickness of the bed; $a=\mathrm{D} / \mathrm{ra}$ is the dimensionless diffusion coefficient; $\mathrm{J}_{0}(\mathrm{z})$ is a zeroorder cylindrical function of the first kind; $z_{i}$ are roots of the equation $d J_{0}(z) / d z=0 ; \Phi$ is an angle in the cylindrical coordinate system.

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